Repetition §5.4, 4.9

$V_1, \dots, V_n$ Unterräume von V.

$V_1 \times \dots \times V_n = \{ (v_1, \dots, v_n) \mid \forall i: v_i \in V_i \}$ Vektorraum

$\quad\; \shortparallel$ direktes Produkt

$V_1 \boxplus \dots \boxplus V_n$ äussere direkte Summe.

$f: V_1 \times \dots \times V_n \longrightarrow W$ lineare Abb. $\quad f(v_1, \dots, v_n) = \underline{f(v_1, 0, \dots, 0)} + \dots + \underline{f(0, \dots, 0, v_n)}$

$$\boxed{\begin{array}{l} f_i : V_i \longrightarrow W \text{ lineare Abb.} \\ \Rightarrow f(v_1, \dots, v_n) := f_1(v_1) + \dots + f_n(v_n) \\ \text{ist linear.} \end{array}}$$

$V_i \hookrightarrow V$, $v_i \longmapsto v_i$.

$\kappa: V_1 \times \dots \times V_n \longrightarrow V$, $(v_1, \dots, v_n) \longmapsto v_1 + \dots + v_n$ linear.

Def.: V ist die innere direkte Summe von $V_1, \dots, V_n$ wenn dies ein Isom. ist.

$V = V_1 \oplus \dots \oplus V_n$.

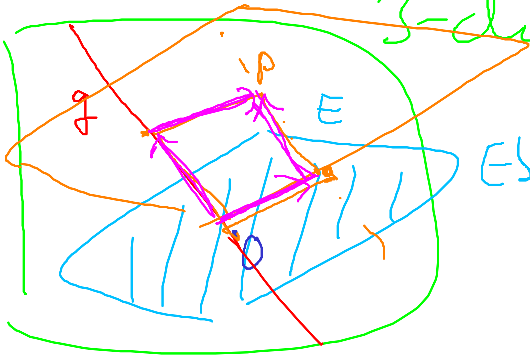
$\underline{n=2}$: $V = V_1 \oplus V_2 \iff \left\{\begin{array}{l} \kappa \text{ surjektiv} \\ \text{und} \\ \kappa \text{ injektiv} \end{array}\right\} \iff \left\{\begin{array}{l} V = V_1 + V_2 \\ \operatorname{Kern}(\kappa) = 0 \end{array}\right\} \iff \left\{\begin{array}{l} V = V_1 + V_2 \\ V_1 \cap V_2 = \{0\} \end{array}\right.$

$V_1 + V_2 = \left\{ v_1 + v_2 \,\middle|\, \begin{array}{l} v_1 \in V_1 \\ v_2 \in V_2 \end{array} \right\}$

$$\kappa(v_1, v_2) = 0 \iff v_1 + v_2 = 0 \iff v_2 = -v_1 \in V_1 \cap V_2$$

$$\Rightarrow \operatorname{Kern}(\kappa) = \{ (v, -v) \mid v \in V_1 \cap V_2 \}$$

3-dim. Raum

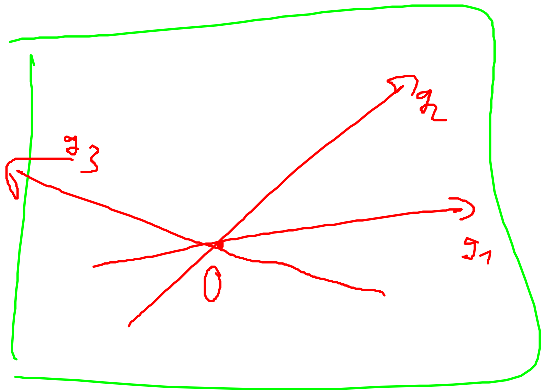


Ebene

$V = E \oplus g$

$3 = 2 + 1$

$g \not\subset E$



$V = g_1 \oplus g_2 \oplus g_3$

$3 = 1 + 1 + 1$

$$\dim(V_1 \oplus V_2) = \dim V_1 + \dim V_2 .$$

I beliebige Menge

V_i Vektorraum für jedes $i \in I$

$$\bigtimes_{i\in I} V_i := \{ (v_i)_{i\in I} \mid \forall i: \ v_i \in V_i \}$$

$\bigtimes_{i\in I} V_i \longrightarrow V_j$ lineare

$(v_i)_i \longmapsto v_j$. Abb.

$$\bigoplus_{i\in I} V_i := \{ (v_i)_{i\in I} \in \bigtimes_{i\in I} V_i \mid \text{fast alle } v_i = 0 \}.$$

K^I

$\cup$

$K^{(I)}$

$$f: \bigoplus_{i\in I} V_i \longrightarrow W, \quad f((v_i)_{i\in I}) = \sum_{i\in I}{}' f\left(\left(\begin{cases} v_i & j=i \\ 0 & j\neq i \end{cases} \right)_{j\in I} \right)$$

Falls $V_i \subset V$ Unterräume.

$V_j \longrightarrow \bigoplus_{i\in I} V_i$ linear,

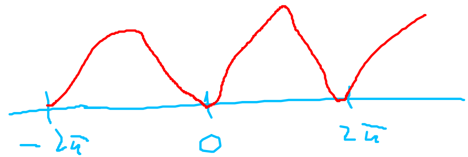
$v_j \mapsto \left(\begin{cases} v_j & i=j \\ 0 & i\neq j \end{cases} \right)_{i\in I}$

$$\rightsquigarrow \bigoplus_{i\in I} V_i \xrightarrow{\kappa} V, \quad (v_i)_{i\in I} \longmapsto \sum_{i\in I} v_i$$

<u>Def.</u>: $V = \bigoplus_{i\in I} V_i$ falls κ ein Isomorphismus ist.

$$V := \{ f: \mathbb{R} \to \mathbb{C} \mid \text{beliebig oft differenzierbar.} \ \forall x \in \mathbb{R}: \forall n \in \mathbb{Z}: f(x + 2\pi n) = f(x) \}.$$

$K = \mathbb{C}$



$$\forall n \in \mathbb{Z}: \quad f_n(x) := e^{inx} \Rightarrow f_n \in V.$$

paarweise verschieden und linear unabhängig.

$$V_n := \langle f_n \rangle$$

$$\bigoplus_{n \in \mathbb{Z}} V_n \hookrightarrow V, \quad (a_n f_n)_{n \in \mathbb{Z}} \mapsto {\sum_{n \in \mathbb{Z}}}' a_n f_n$$

$$a_n \in \mathbb{C}$$

$$\boxed{e^{ix} = \cos x + i \sin x}$$

X_i Mengen:

$$\bigtimes_{i \in I} X_i = \{ \text{Abb. } I \to \bigcup_{i \in I} X_i \mid \forall i: x_i \in X_i \}$$

$$i \mapsto x_i$$

$$\boxed{(x_i)_{i \in I}}$$

$$K^n = \{ \text{Abb. } \{1, \dots, n\} \to K \}$$
$$= \{ (x_1, \dots, x_n) \mid \text{alle } x_i \in K \}$$